NAG Toolbox for MATLAB

s10ab

1 Purpose

s10ab returns the value of the hyperbolic sine, $\sinh x$, via the function name.

2 Syntax

```
[result, ifail] = s10ab(x)
```

3 Description

s10ab calculates an approximate value for the hyperbolic sine of its argument, $\sinh x$.

For $|x| \leq 1$ it uses the Chebyshev expansion

$$\sinh x = x \times y(t) = x{\sum_{r=0}}' a_r T_r(t)$$

where $t = 2x^2 - 1$.

For $1 < |x| \leq E_1$, $\quad \sinh x = \frac{1}{2}(e^x - e^{-x})$

where E_1 is a machine-dependent constant.

For $|x| > E_1$, the function fails owing to the danger of setting overflow in calculating e^x. The result returned for such calls is $\sinh(\operatorname{sign} xE_1)$, i.e., it returns the result for the nearest valid argument.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

The function has been called with an argument too large in absolute magnitude. There is a danger of setting overflow. The result is the value of sinh at the closest argument for which a valid call could be made.

7 Accuracy

If δ and ϵ are the relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq |x \coth x \times \delta|.$$

That is the relative error in the argument, x, is amplified by a factor, approximately $x \coth x$. The equality should hold if δ is greater than the ***machine precision*** (δ is a result of data errors etc.) but, if δ is simply a result of round-off in the machine representation of x, then it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the error amplification factor can be seen in the following graph:

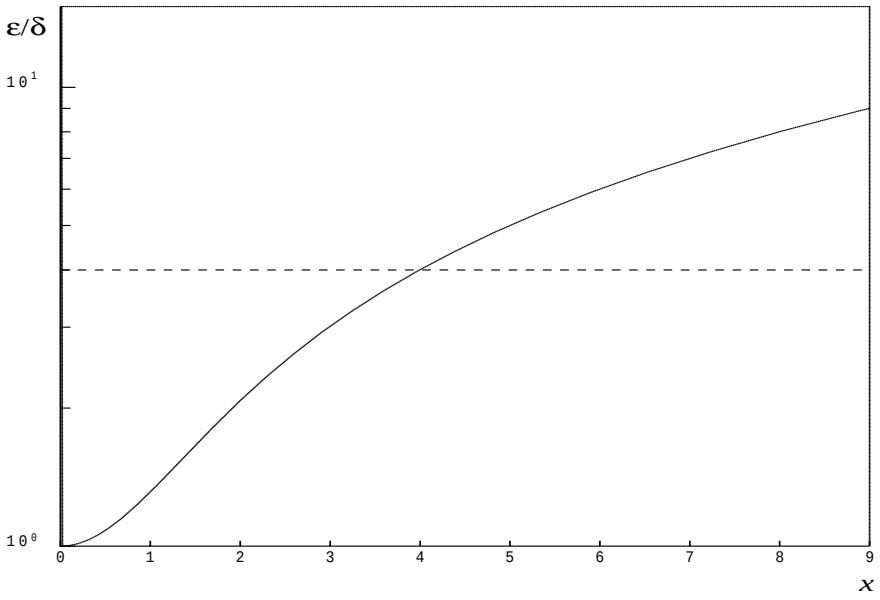

Figure 1

It should be noted that for $|x| \geq 2$

$$\epsilon \sim x\delta = \Delta$$

where Δ is the absolute error in the argument.

8 Further Comments

None.

9 Example

```
x = -10;
[result, ifail] = s10ab(x)
```

```
result =
  -1.1013e+04
ifail =
           0
```